JEE Main Exam 2022 - Session 2

27 Jul 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. The activity of radioactive material is 6.4×10^{-4} Curie. Its half life is 5 days, then the activity is 5×10^{-6} Curie after

A) 7 days

B) 15 days

C) 25 days

D) 35 days

Answer: 35 days

Solution: Given that, $A_0 = 6.4 \times 10^{-4}$ Curie

$A = 5 \times 10^{-6}$ Curie

The activity of a radioactive sample at time t is given by,

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$$

$$\Rightarrow \frac{5\times10^{-6}}{6.4\times10^{-4}} = \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

$$\Rightarrow 7 = \frac{t}{5}$$

$$\Rightarrow t = 35 \text{ days}$$

Q.2. Block moves down an inclined rough plane with constant velocity. The contact force experienced by the block is (Mass of block is M)

A) Mg

B) $Mg\sin\theta$

C) $Mg(\sin\theta + \cos\theta)$

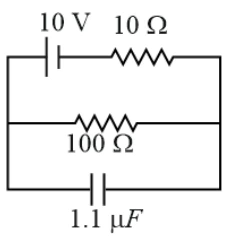
D) $\sqrt{Mg}$

Answer: Mg

Solution: As the block moves with constant velocity, so force on the block should be balanced. The forces acting on the block are its weight, normal contact force and friction. The contact force is vector sum of normal contact and friction. As the forces are balanced, the contact force should be equal and opposite to its weight.

$\therefore$Contact force $= Mg$

Q.3. For the given circuit, the charge on capacitor in steady state is $N \times 10^{-6}$ C. The value of N is ______



A) 10

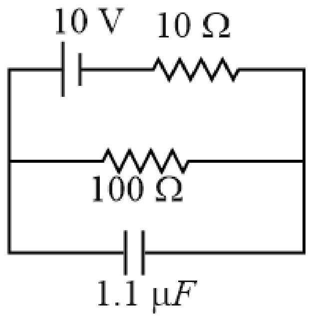
B) 5

C) 15

D) 20

Answer: 10

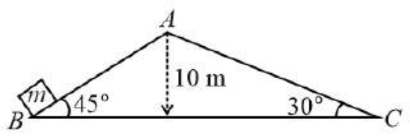
Solution:



At the steady state, there will be no current through the capacitor. Therefore, potential difference across capacitor $V_c = \left(\frac{10}{110}\right) \times 100\ \text{V}$.

Charge on capacitor $q = CV_c = \left(1.1 \times 10^{-6}\right) \times \frac{1000}{110} = 10 \times 10^{-6}\ \text{C}$

Q.4. A block of mass m is pushed above the plane from B such that it just crossed A and moves to C. Ignore friction everywhere, if time taken to move from B to A, then to C is $k\left(\sqrt{2}+1\right)$ s, then the value of k is, $\left(\text{Take, } g = 10\ \text{m s}^{-2}\right)$



A) 2

B) 1

C) 3

D) 0.5

Answer: 2

Solution: By energy conservation $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_C^2$

And $v_B = \sqrt{2gh} = 10\sqrt{2}\ \text{m s}^{-1}$

Acceleration of a particle moving on a smooth incline is $g\sin\theta$. Therefore,

Along BA, $a_{BA} = g\sin 45^\circ = 5\sqrt{2}\ \text{m s}^{-2}$. Using equation of motion,

$v = u + at$
$\Rightarrow 0 = 10\sqrt{2} - 5\sqrt{2}t_{BA}$
$\Rightarrow t_{BA} = 2\ \text{s}$

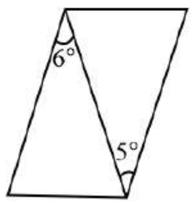
Similarly along AC

$$t_{AC} = \frac{10\sqrt{2}}{g\sin 30^\circ} = \frac{10\sqrt{2}}{5} = 2\sqrt{2}\ \text{s}$$

$\Rightarrow$Total time $= 2\sqrt{2} + 2 = 2\left(\sqrt{2}+1\right)$ s.

Therefore, $k = 2$

Q.5. A prism of angle 6° and refractive index 1.5 and another prism of angle 5° and refractive index 1.55 are merged together as shown. If the angle of deviation is $\left(\frac{1}{x}\right)^\circ$, then the value of x is _______.



A) 4

B) 2

C) 3

D) 1

Answer: 4

Solution: The formula for deviation due to a thin prism is given by, $\delta = A(\mu - 1)$.

Therefore, for prism 1:

$\delta_1 = 6(1.5 - 1) = 3^\circ$

And for prism 2:

$\delta_2 = -5(1.55 - 1) = -2.75^\circ$

$\Rightarrow \delta_{\text{net}} = \delta_1 + \delta_2$
$= 3^\circ - 2.75^\circ$
$= 0.25^\circ$
$= \left(\frac{1}{4}\right)^\circ$
$\Rightarrow \left(\frac{1}{4}\right)^\circ$

Therefore, $x = 4$

Q.6. Energy Density U can be given as:

$U = \frac{\alpha}{\beta}\sin\frac{\alpha x}{kt}$, where x is length, t is temperature, k is Boltzmann constant, α and β are constants, then the dimension of β is

A) $\mathrm{M^0L^0T^0}$

B) $\mathrm{M^0L^2T^{-2}}$

C) $\mathrm{M^1L^2T^{-2}}$

D) $\mathrm{M^0L^2T^0}$

Answer: $\mathrm{M^0L^2T^0}$

Solution: $U = \frac{\alpha}{\beta}\sin\frac{\alpha x}{kt}$

Dimensions of αx and $k t$ should be same

$[\alpha][x] = [k][t] \quad \Rightarrow [\alpha] = \frac{[k][t]}{[x]}$

$[U] = \frac{[\alpha]}{[\beta]}$

$\Rightarrow [\beta] = \frac{[\alpha]}{[U]} = \frac{[k][t]}{[x]\left[\frac{E}{V}\right]} = \frac{\left[\frac{E}{t}\right][t]}{[x]\left[\frac{E}{V}\right]}$

$\Rightarrow [\beta] = \frac{\left[L^3\right]}{[L]} = \left[\mathrm{M^0L^2T^0}\right]$

Q.7. If a charge $4\mu\mathrm{C}$ is divided into two and kept apart at some distance. The magnitude of charges so that the force between them is maximum is

A) $1\mu\mathrm{C}$ and $3\mu\mathrm{C}$

B) $2\mu\mathrm{C}$ and $2\mu\mathrm{C}$

C) $0\mu\mathrm{C}$ and $4\mu\mathrm{C}$

D) $1.5\mu\mathrm{C}$ and $2.5\mu\mathrm{C}$

Answer: $2\mu\mathrm{C}$ and $2\mu\mathrm{C}$

Solution: Let us assume that charges are divided as x & $(4 - x)$.

Then force will be given by,

$F = \frac{k(4-x)(x)}{a^2}$

For F to be maximum:

$\frac{dF}{dx} = 0$

$\Rightarrow 4 - 2x = 0$

$\Rightarrow x = 2$

Therefore, the charges should be $2\ \mu\mathrm{C}$ and $2\ \mu\mathrm{C}$

Q.8. An object is thrown vertically upwards with velocity equal to $\lambda v_e\ (\lambda < 1)$ from the surface of the earth. Maximum height achieved by the object from surface of earth is

A) $\frac{Re}{1+\lambda^2}$

B) $\frac{\left(1+\lambda^2\right)Re}{1-\lambda^2}$

C) $\frac{\lambda^2 Re}{1-\lambda^2}$

D) $\left(1-\lambda^2\right)Re$

Answer: $\frac{\lambda^2 Re}{1-\lambda^2}$

Solution: Escape velocity is given by, $v_e=\sqrt{\frac{2GMe}{Re}}$

Using mechanical energy conservation,

$\Rightarrow \frac{1}{2}m\left(\lambda\sqrt{\frac{2GMe}{Re}}\right)^2-\frac{GMem}{Re}=-\frac{GMem}{Re+h}$

$\Rightarrow \lambda^2\frac{GMem}{Re}-\frac{GMem}{Re}=-\frac{GMem}{Re+h}$

$\Rightarrow (Re+h)=\frac{Re}{1-\lambda^2}$

$\Rightarrow h=\frac{\lambda^2 Re}{1-\lambda^2}$

Q.9. A proton $\left(m=1.6\times10^{-27}\ \mathrm{kg}\right)$ moves in a circle of radius $60\ \mathrm{cm}$ in uniform magnetic field $1\ \mathrm{T}$ in transverse direction. Kinetic energy of the proton (in MeV) is equal to

A) 18

B) 12

C) 10

D) 6

Answer: 18

Solution: Kinetic energy of the moving particle in uniform magnetic field is given by, $KE=\frac{B^2q^2r^2}{2\mathrm{m}}$

$=\frac{1^2\left(1.6\times10^{-19}\right)^2(0.6)^2}{2\times\left(1.6\times10^{-27}\right)}$

$=0.18\times\left(1.6\times10^{-19}\right)\times10^8\ \mathrm{J}$

$=0.18\times10^8\ \mathrm{eV}$

$=18\ \mathrm{MeV}$

Q.10. In an amplitude modulation, amplitude of carrier wave is $5\ \mathrm{V}$ while amplitude of modulating signal is $1\ \mathrm{V}$. The modulation index of message signal is

A) 10%

B) 20%

C) 15%

D) 30%

Answer: 20%

Solution: Modulation index of message signal is given by,

$\mu=\frac{Am}{Ac}$

$=\frac{1}{5}$

$=20\%$

Q.11. Two waves of same frequency having intensity k and $4k$ interfere with each other. The ratio of maximum to minimum intensity in interference pattern is

A) $9:1$

B) $3:1$

C) $5:1$

D) $5:3$

Answer: $9:1$

Solution: Intensity is given by, $I=I_1+I_2+2\sqrt{I_1}\sqrt{I_2}\cos\theta$.

Hence, we can write

$$\Rightarrow \frac{I\max}{I\min}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$$

$$=\frac{9}{1}$$

Q.12. In common-emitter configuration when base current is varied from $20\mu\text{A}$ to $25\mu\text{A}$, the collector current varies from $6\ \text{mA}$ to $8\ \text{mA}$. The current gain factor is equal to

A) 150

B) 200

C) 400

D) 40

Answer: 400

Solution: Current gain: It is the ratio of change in output current divided by change in input current. For common CE (Common Emitter) configuration, it is the ratio of change in collector current divided by change in base current. Therefore,

$$\beta=\frac{\Delta I_C}{\Delta I_B}$$
$$=\frac{2\text{ mA}}{5\,\mu A}$$
$$=\frac{2000}{5}$$
$$=400$$

Q.13. A dielectric $(\varepsilon_r=k)$ is used to fill the space between plates of a capacitor of capacitance C. The new capacitance of capacitor is

A) $\frac{C}{k}$

B) $(k-1)C$

C) kC

D) k^2C

Answer: kC

Solution: As we know, $C_0=\frac{\varepsilon_0\varepsilon_r A}{d}$

When, $\varepsilon_r=1$

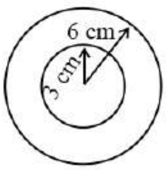
$$C_0=\frac{\varepsilon_0 A}{d}=C$$

When, $\varepsilon_r=k$

$$C_0'=\frac{\varepsilon_0 A}{d}k$$

$$=kC$$

Q.14. Soap bubble $6\ \text{cm}$ is enclosing another bubble of radius $3\ \text{cm}$. The inside bubble is experiencing an internal pressure. The same internal pressure is experienced by another bubble of radius R outside. Find the value of R in cm.



A) $2\ \text{cm}$

B) $3\ \text{cm}$

C) $4\ \text{cm}$

D) $5\ \text{cm}$

Answer: $2\ \text{cm}$

Solution: Pressure inside a bubble is given by, $P_0 + \frac{4T}{r}$.

Pressure inside the bubble of smaller radius will be,

$\left[\left(P_0 + \frac{4T}{6}\right) + \frac{4T}{3}\right]$

Pressure inside the bubble with radius R, $P_0 + \frac{4T}{R}$

Now,

$\Rightarrow P_0 + \frac{4T}{6} + \frac{4T}{3} = P_0 + \frac{4T}{R}$

$\Rightarrow \frac{1}{R} = \frac{1}{6} + \frac{1}{3}$

$\Rightarrow R = 2\ \text{cm}$

Q.15. For an EM wave chose the correct option regarding following statements (E denotes the electric field component, B denotes the magnetic field component and c denotes the velocity of EM wave at a point.

(a) $\vec{E} \perp \vec{B}$, $\vec{E} \| \vec{c}$

(b) $\vec{E} \perp \vec{B}$

(c) $\vec{E} \perp \vec{c}$

(d) $\hat{E} = \hat{c} \times \hat{B}$

A) Only a and c are true

B) Only a and b are true

C) Only b, c, d are true

D) Only b and c are true

Answer: Only b and c are true

Solution: In electromagnetic waves, the oscillations of E and B are perpendicular to each other and are in same phase and they travel through vacuum with same speed.

Therefore,

$\Rightarrow \vec{E}$, $\vec{B}$ & $\vec{c}$ are mutually perpendicular such that $\hat{B} \times \hat{c} = \hat{E}$.

So only b and c are correct.

Q.16. In photoelectric effect

a) Photoelectric current increases by increasing incident frequency

b) Photoelectric current increases by increasing incident intensity

c) Stopping potential increases in magnitude by increasing incident frequency

d) Stopping potential increases in magnitude by increasing incident intensity

A) Only a and b are true

B) Only a and c are true

C) Only b and d are true

D) Only $b$and c are true

Answer: Only $b$and c are true

Solution: As stopping potential is a measure of the kinetic energy of the electron, which comes from the energy of incident light, therefore, stopping potential increases with increase in frequency.

If we increase the intensity of the incident radiation by keeping the frequency constant, then the value of saturation current increases in proportion to the intensity of the incident radiation. Thus, saturation Current increases with increases in intensity

Section B: Chemistry

Q.17. Assertion: Boron is unable to form BF_6^{3-}.

Reason: Boron is very small in size.

A) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion

B) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion

C) Assertion is correct but Reason is incorrect

D) Assertion is incorrect but Reason is correct

Answer: Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion

Solution: Due to non-availability of d orbitals, boron is unable to expand its octet. Therefore, the maximum covalence of boron cannot exceed 4.

Boron is the first element from top in group 13 and the atomic size increases down the group. Therefore, boron is smallest in size.

Q.18. Which of the following species is formed when $KMnO_4$ reacts with $S_2O_3^{2-}$ in alkaline medium?

A) SO_4^{2-}

B) SO_3^{2-}

C) $S_2O_7^{2-}$

D) $S_2O_8^{2-}$

Answer: SO_4^{2-}

Solution: Thiosulphate is oxidised almost quantitatively to sulphate:

$$8\,MnO_4{}^{-} + 3S_2O_3{}^{2-} + H_2O \longrightarrow 8\,MnO_2 + 6\,SO_4{}^{2-} + 2\,OH^{-}$$

Q.19. Match column I with column II.

	Polymer		Monomer
(A)	Neoprene	(1)	Acrylonitrile
(B)	Natural rubber	(2)	Chloroprene
(C)	Teflon	(3)	Isoprene
(D)	Orlon	(4)	Tetrafluoroethene

A) (A)-(2), (B)-(3), (C)-(4), (D)-(1)

B) (A)-(1), (B)-(2), (C)-(3), (D)-(4)

C) (A)-(4), (B)-(3), (C)-(1), (D)-(2)

D) (A)-(3), (B)-(1), (C)-(4), (D)-(2)

Answer: (A)-(2), (B)-(3), (C)-(4), (D)-(1)

Solution: Neoprene (CR), also called polychloroprene or chloroprene rubber, synthetic rubber produced by the polymerization (or linkage together of single molecules into giant, multiple-unit molecules) of chloroprene.

Natural rubber is polyisoprene. It is a polymer of hydrocarbon-isoprene. Polyisoprene is an addition polymer. It is 2-methyl-$1,\ 3$-butadiene or cis-$1,\ 4$-polyisoprene.

Teflon is a brand name for a synthetic chemical called polytetrafluoroethylene (PTFE). Orlon is made from polymerized acrylonitrile.

Q.20. $100\ \mathrm{mL}$ of $0.1\ \mathrm{M} H_2SO_4$ is reacted with $50\ \mathrm{mL}$ of $0.1\ \mathrm{M}$ of NaOH. What is the normality of H_2SO_4 left in the solution?

A) $0.2\ \mathrm{N}$

B) $0.1\ \mathrm{N}$

C) $0.4\ \mathrm{N}$

D) $7.5\ \mathrm{N}$

Answer: $0.1\ \mathrm{N}$

Solution:

$$\begin{array}{lll} & H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O & \\ \text{Initial} & 10\ \mathrm{m.mol} \quad 5\ \mathrm{m.mol} & \\ \text{Final} & 10-2.5 & \\ & =7.5\ \mathrm{m.mol} & 2.5\ \mathrm{m.mol} \end{array}$$

$$\begin{aligned} \text{Normality of } H_2SO_4 &= \text{Molarity of } H_2SO_4 \times n_f \\ &= \frac{7.5}{150} \times 2 = 0.1\ \mathrm{N} \end{aligned}$$

Q.21. Consider the following orbitals (A to D) containing electrons with following set of quantum numbers (n, l, m, s).

1. $3, 2, 1, \frac{1}{2};$
2. $4, 0, 0, -\frac{1}{2};$
3. $3, 1, 1, \frac{1}{2};$
4. $3, 0, 0, \frac{1}{2};$

The highest energy orbital among the above set of orbitals for a multielectron atom will be:

A) 1

B) 2

C) 3

D) 4

Answer: 1

Solution: 1. $3d$

2. $4s$

3. $3p$

4. $3s$

Same orbitals will have same energy and higher the value of $(n + l)$ higher is the energy, Their energies will be in order:

$1 > 2 > 3 > 4$

Q.22. Match Column I with Column II.

	Column I		Column II
1.	$\Psi_{AB} = \Psi_A + \Psi_B$	P.	Bonding molecular orbital
2.	$\mu = q \times d$	Q.	Antibonding molecular orbital
3.	$\Psi^*_{AB} = \Psi_A - \Psi_B$	R.	Dipole moment
4.	$\frac{\text{number of bonding } e^- - \text{number of antibonding } e^-}{2}$	S.	Bond order

A) 1-P, 2-R, 3-Q, 4-S

B) 1-P, 2-Q, 3-R, 4-S

C) 1-Q, 2-P, 3-R, 4-S

D) 1-R, 2-P, 3-Q, 4-S

Answer: 1-P, 2-R, 3-Q, 4-S

Solution: Mathematically, the formation of molecular orbitals may be described by the linear combination of atomic orbitals that can take place by addition and by subtraction of wave functions of individual atomic orbitals as shown below:

$\psi_{MO} = \psi_A \pm \psi_B$

Therefore, the two molecular orbitals σ and σ^* are formed as:

$\sigma = \psi_A + \psi_B$

$\sigma^* = \psi_A - \psi_B$

The molecular orbital σ formed by the addition of atomic orbitals is called the bonding molecular orbital while the molecular orbital σ^* formed by the subtraction of atomic orbital is called antibonding molecular orbital

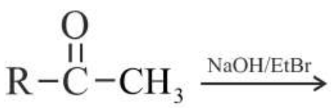
Bond order (b.o.) is defined as one half the difference between the number of electrons present in the bonding and the antibonding orbitals i.e.,

Bond order (b.o.) $= \frac{1}{2}(N_b - N_a)$

possesses the dipole moment (depicted below) which can be defined as the product of the magnitude of the charge and the distance between the centres of positive and negative charge. It is usually designated by a Greek letter 'μ'. Mathematically, it is expressed as follows :

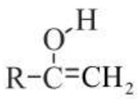
Dipole moment $(\mu) =$ charge $(Q) \times$ distance of separation (r)

Q.23. What is the final product of the given reaction?

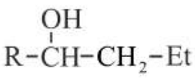


A)

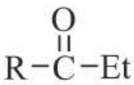
B)



C)

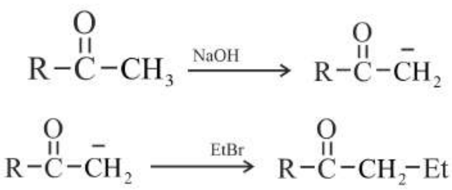


D)



Answer:

Solution: Base pick the alpha hydrogen of the given ketone. The carbanion formed undergo nucleophilic substitution reaction with ethyl bromide.



Q.24. Lathering property of soap is due to which of the following?

A) Sodium stearate

B) Sodium carbonate

C) Sodium rosinate

D) Glycerol

Answer: Sodium rosinate

Solution: Shaving soaps contain glycerol to prevent rapid drying. A gum called, rosin is added while making them. It forms sodium rosinate which lathers well. Laundry soaps contain fillers like sodium rosinate, sodium silicate, borax and sodium carbonate.

Q.25. Statement I: On dilution, molar conductivity for KI(aq.) increases steeply.

Statement II: On dilution, molar conductivity for carbonic acid (aq.) slowly increases till infinite dilution.

A) Statement I is correct and Statement II is incorrect

B) Both Statement I and Statement II are correct

C) Statement I is incorrect and Statement II is correct

D) Both Statement I and Statement II are incorrect

Answer: Both Statement I and Statement II are incorrect

Solution: For strong electrolytes, $\wedge_m$ increases slowly with dilution and can be represented by the equation:

$$\wedge_m = \wedge_m^o - A\ c^{\frac{1}{2}}$$

It can be seen that if we plot $\wedge_m$ against $c^{\frac{1}{2}}$, We obtain a straight line with intercept equal to $\wedge_m$ and slope equal to '$-A$'. The value of the constant 'A' for a given solvent and temperature depends on the type of electrolyte. KI is a strong electrolyte.

Weak electrolytes like carbonic acid and have lower degree of dissociation at higher concentrations and hence for such electrolytes, the change in $\wedge_m$ with dilution is due to increase in the degree of dissociation and consequently the number of ions in total volume of solution that contains $1\ \text{mol}$ of electrolyte. In such cases $\wedge_m$ increases steeply on dilution, especially near lower concentrations. Therefore, $\wedge_m^o$ cannot be obtained by explanation of $\wedge_m$ to zero concentration.

Q.26. Consider a first order reaction:

$$A \longrightarrow B$$

The concentration of A after 70 minutes becomes half. If the rate constant of the reaction is $x \times 10^{-6}\ \text{second}^{-1}$, then find the value of x. [Take $\ln\ 2 = 0.693$]

A) 693

B) 420

C) 165

D) 330

Answer: 165

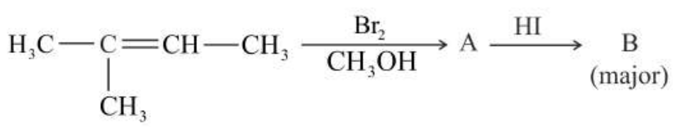
Solution: For first order reaction:

$$t_{\frac{1}{2}} = \frac{0.693}{k}$$

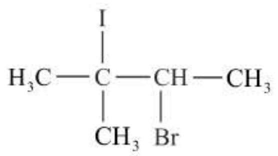
$$k = \frac{0.693}{70 \times 60} = 0.000165\ s^{-1}$$

$$= 165 \times 10^{-6}\ s^{-1}$$

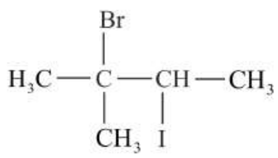
Q.27. The product B is:



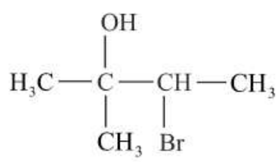
A)



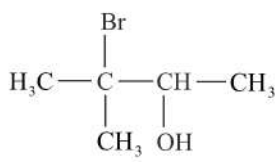
B)



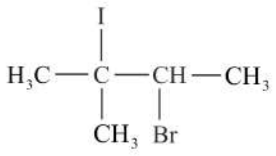
C)



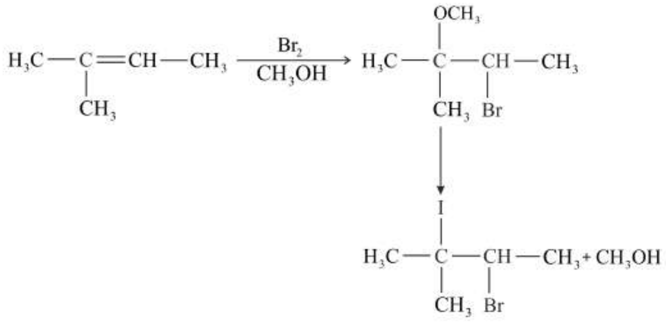
D)



Answer:



Solution:



Q.28. Match Column I with Column II.

Column I		Column II	
A.	Imp. Aniline$+$Water	P.	Crystallisation
B.	Aniline$+$Chloroform	Q.	Steam distillation followed by use of separating funnel
C.	Benzoic acid$+$Naphthalene	R.	Sublimation
D.	Naphthalene$+$Non volatile salt	S.	Distillation

A) A-Q, B-S, C-P, D-R

B) A-P, B-Q, C-R, D-S

C) A-S, B-R, C-Q, D-P

D) A-Q, B-P, C-R, D-S

Answer: A-Q, B-S, C-P, D-R

Solution: Aniline is purified using the process of steam distillation because aniline is steam volatile and it is insoluble in water.

Chloroform and aniline having boiling points $334\ \mathrm{K}$ and $457\ \mathrm{K}$ can be separated by distillation. More volatile chloroform vaporizes first and is collected in the receiver. Aniline is left in the distillation flask.

Benzoic acid is soluble in hot water but naphthalene is not soluble, hence separation can be carried out by hot water crystallisation.

Naphthalene can also be separated by sublimation from non-volatile at room temperature.

The vapor of naphthalene can be collected and solidified.

Q.29. How many among the following ores contain Fe(iron)?

Siderite, Malachite, Magnetite, Haematite, Cryolite, Cuprite, Limonite, Kaolinite, Sphalerite, Bauxite, Chalcopyrite.

A) 6

B) 5

C) 2

D) 3

Answer: 5

Solution: Siderite: $FeCO_3$

Malachite: $CuCO_3 . Cu(OH)_2$

Magnetite: Fe_3O_4

Haematite: Fe_2O_3

Cryolite: Na_3AlF_6

Cuprite: Cu_2O

Limonite: $FeO(OH) . nH_2O$

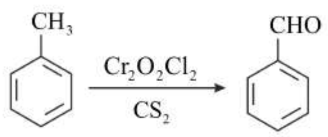
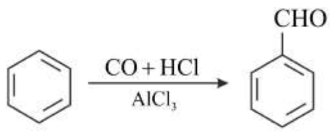
Kaolinite: $[Al_2(OH)_4Si_2O_5]$

Sphalerite: Zns

Bauxite: $AlO_x(OH)_{3-2x}$

Chalcopyrite: $CuFeS_2$

Q.30. Match the reactions given in Column I with their corresponding names given in Column II.

	Column I		Column II
1.		a.	Etard's reaction
2.		b.	Gattermann Koch reaction
3.	$R-CN \xrightarrow[H_2O]{SnCl_2+HCl} R-CHO$	c.	Stephen's Reduction
4.	$R-COCl \xrightarrow[H_2]{Pd-BaSO_4} R-CHO$	d.	Rosenmund reduction

A) 1-a, 2-b, 3-c, 4-d

B) 1-b, 2-a, 3-c, 4-d

C) 1-b, 2-c, 3-a, 4-d

D) 1-c, 2-b, 3-a, 4-d

Answer: 1-a, 2-b, 3-c, 4-d

Solution: Acyl chloride (acid chloride) is hydrogenated over catalyst, palladium on barium sulphate. This reaction is called Rosenmund reduction.

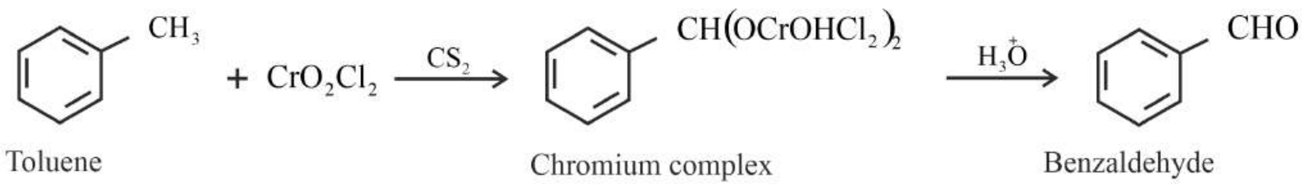
$$R-COCl \xrightarrow[Pd-BaSO_4]{H_2} R-CHO$$

Nitriles are reduced to corresponding imine with stannous chloride in the presence of hydrochloric acid, which on hydrolysis give corresponding aldehyde.

$$RCN + SnCl_2 + HCl \longrightarrow RCH = NH \xrightarrow{H_2O} RCHO$$

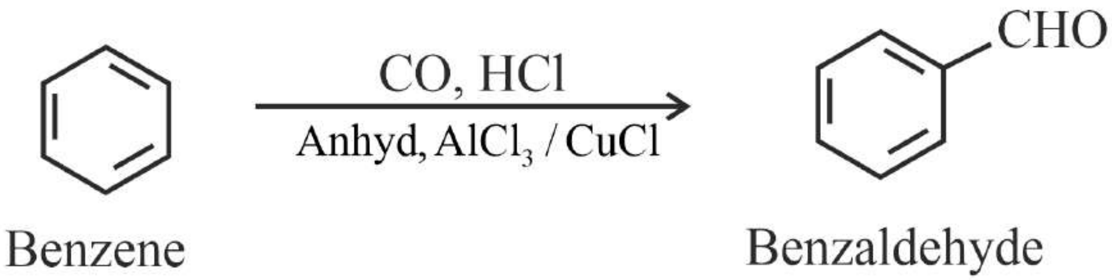
This reaction is called Stephen reaction.

Use of chromyl chloride (CrO_2Cl_2): Chromyl chloride oxidises methyl group to a chromium complex, which on hydrolysis gives corresponding benzaldehyde.



This reaction is called Etard reaction.

When benzene or its derivative is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous aluminium chloride or cuprous chloride, it gives benzaldehyde or substituted benzaldehyde.



This reaction is known as Gatterman-Koch reaction.

Q.31. $5.0\ g$ of toluene is subjected to controlled oxidation to get benzaldehyde. The percentage yield of the product formed in the above reaction is 92%. Find the mass of benzaldehyde formed in g.

A) $5.3\ g$

B) $10.6\ g$

C) $9.2\ g$

D) $2.5\ g$

Answer: $5.3\ g$

Solution: $\underset{\text{Toluene}}{C_6H_5(CH_3)} \xrightarrow{\text{Oxidation}} \underset{\text{Benzaldehyde}}{C_6H_5CHO}$

Molar mass of benzaldehyde=$106\ g/mol$

No of moles of Benzaldehyde $= 0.92 \times \frac{5}{92}$

Mass of Benzaldehyde $= 0.92 \times \frac{5}{92} \times 106 = 5.30\ g$

Q.32. Consider the following reaction:

$Fe^{3+} + A \rightarrow$Prussian blue

Which of the following represent A?

A) A is $[Fe(CN)_6]^{4-}$

B) A is $[Fe(CN)_6]^{3-}$

C) A is $[FeCl_4]^-$

D) A is $FeSO_4$

Answer: A is $[Fe(CN)_6]^{4-}$

Solution: $Fe^{3+} + [Fe(CN)_6]^{4-} \rightarrow \underset{\text{(Prussian blue)}}{Fe_4[Fe(CN)_6]_3}$

Q.33. Correct order of first ionisation energy for the elements with given electronic configuration is:

(I) $3s^2$

(II) $3s^2 3p^1$

(III) $3s^2 3p^3$

(IV) $3s^2 3p^4$

A) (III) > (IV) > (I) > (II)

B) (IV) > (III) > (II) > (I)

C) (I) > (II) > (III) > (IV)

D) (II) > (III) > (I) > (IV)

Answer: (III) > (IV) > (I) > (II)

Solution: Electronic configuration of $Al = 1s^2 2s^2 2p^6 3s^2 3p^1$

Electronic configuration of $Mg = 1s^2 2s^2 2p^6 3s^2$

Al has one unpaired electron in its highest energy orbital $(3p)$, and Mg's highest energy orbital $(3s)$ has the paired electrons.

It is energetically favourable for all the electrons in an orbital to be paired, which means that breaking up this pair would require more energy.

Compared to $16th$ group, 15 group have high ionisation energy. It is because of the stable half filled configuration of he $15th$ group elements.

$P - 3s^2 3p^3$

$S - 3s^2 3p^4$

Therefore P has more ionisation energy than S

Q.34. Group-I element (A) with maximum hydration enthalpy, shows similarity with group-II element (B). (A) and (B) respectively are:

A) $Li,\ Mg$

B) $Be,\ Mg$

C) $Na,\ Ca$

D) $K,\ Be$

Answer: $Li,\ Mg$

Solution: Lithium-ion has the highest hydration enthalpy and shows similarity with magnesium ion. The similarity between lithium and magnesium is particularly and arises because of their similar sizes : atomic radii, $Li = 152\ pm,\ Mg = 160\ pm$; ionic radii : $Li^{+} = 76\ pm,\ Mg^{2+} = 72\ pm.$

Section C: Mathematics

Q.35. Let $A=\begin{bmatrix}4 & -2\\ \alpha & \beta\end{bmatrix}$. If $A^2+\gamma A+18I=0$, then $\det(A)$ equals

A) -18

B) 18

C) -50

D) 50

Answer: 18

Solution: Given $A=\begin{bmatrix}4 & -2\\ \alpha & \beta\end{bmatrix} \Rightarrow \det(A)=4\beta+2\alpha$

Characteristic equation of matrix:

$\begin{bmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{bmatrix}=0$

$\Rightarrow 4\beta+\lambda^2-(\beta+4)\lambda+2\alpha=0$

$\Rightarrow \lambda^2-(\beta+4)\lambda+2\alpha+4\beta=0$

Comparing with given equation $A^2+\gamma A+18I=0$, we get

$\Rightarrow \gamma=-(\beta+4)\ \&\ 2\alpha+4\beta=18$

Hence $\det(A)=4\beta+2\alpha=18$

Q.36. The area of region enclosed by $y\le 4x^2, x^2\le 9y, y\le 4$ is equal to:

A) $\frac{40}{3}$

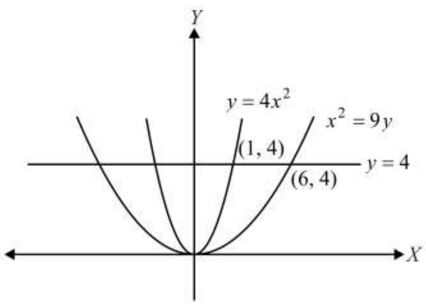
B) $\frac{56}{3}$

C) $\frac{112}{3}$

D) $\frac{80}{3}$

Answer: $\frac{80}{3}$

Solution: Plotting the required region



Enclosed area $=2\int_0^4\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy$

$=2\int_0^4\frac{5\sqrt{y}}{2}dy$

$=5\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$

$=\frac{10}{3}\left(4\right)^{\frac{3}{2}}=\frac{80}{3}$

Q.37. If the length of the latus rectum of a parabola whose focus is (a,a) and tangent at its vertex is $x+y=a$, is 16 . Then $|a|$ is equal to:

A) $2\sqrt{3}$

B) $2\sqrt{2}$

C) $4\sqrt{2}$

D) 4

Answer: $4\sqrt{2}$

Solution: Given, tangent at vertex is $x+y=a$

Now, we know that length of perpendicular from focus (a,a) to tangent at vertex will be given by,

$l=\left|\frac{a+a-a}{\sqrt{1^2+1^2}}\right|$

$\Rightarrow l=\left|\frac{a}{\sqrt{2}}\right|$

So length of latus rectum will be, $4l=16$

$\Rightarrow\ 2\sqrt{2}|a|=16$

$\Rightarrow\ |a|=4\sqrt{2}$

Q.38.

Let $f(x)=\frac{(729p(1+x))^{\frac{1}{7}}-3}{(729(1+qx))^{\frac{1}{3}}-9}$, and $f(x)$ is continuous at $x=0$, then:

A) $21qf(0)-p=0$

B) $21q^2(0)-p^3=0$

C) $21p^2f(0)-q^3=0$

D) $p^2f(0)7q^2=0$

Answer: $21qf(0)-p=0$

Solution:

Given, $f(x)=\frac{(729p(1+x))^{\frac{1}{7}}-3}{(729(1+qx))^{\frac{1}{3}}-9}$

Now, $\lim_{x\to0}f(x)$ exists if numerator of $f(x)$ is zero at $x=0$, its only possible when $p=3$

Now, $\lim_{x\to0}f(x)=\lim_{x\to0}\frac{3\left[\left(x+1\right)^{\frac{1}{7}}-1\right]}{9\left[\left(1+qx\right)^{\frac{1}{3}}-1\right]}$

Now by using binomial approximation we get,

$\lim_{x\to0}f(x)=\lim_{x\to0}\frac{3\left[\left(x+1\right)^{\frac{1}{7}}-1\right]}{9\left[\left(1+qx\right)^{\frac{1}{3}}-1\right]}=\frac{1}{3}\left(\frac{\frac{1}{7}}{\frac{q}{3}}\right)=\frac{1}{7q}=f(0)$

So, $21qf(0)=21q\times\frac{1}{7q}=3=p$

$\Rightarrow\ 21qf(0)-p=0$

Q.39.

If $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$ and $\frac{|\mathrm{adj}(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}(A))))|}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\cdot3^{16}$ where α,β,γ are distinct natural number, then number of possible triplets of (α,β,γ) is ______ .

A) 55

B) 50

C) 40

D) 30

Answer: 55

Solution: $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$

Now $R_3\to R_3+R_1$

$A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma\end{bmatrix}$

$\Rightarrow \quad |A|=(\alpha+\beta+\gamma)\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

We know $\left|\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}\left(A\right)\right)\right)\right)\right|=\left|A\right|^{(2)^4}=\left|A\right|^{16}$

$=(\alpha+\beta+\gamma)^{16}(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}$

Clearly $(\alpha+\beta+\gamma)^{16}=2^{32}\cdot 3^{16}=(12)^{16}$

$\Rightarrow \quad \alpha+\beta+\gamma=12$

Number of possible triplets $={}^{12-1}C_{3-1}=55$

Q.40. Let $f(x)=\min\{[x],[x-1],[x-2],\cdots,[x-10]\}$ where [.] denotes greatest integer function. Then $\int_0^{10}\left(f(x)+|f(x)|+f^2(x)\right)dx$ is equal to:

A) 55

B) 385

C) 5050

D) 270

Answer: 385

Solution: $\forall x\in(0,10)$ we know $f(x)=[x-10]$

Also $f(x)\le 0$

So, $\int_0^{10}(f(x)+|f(x)|)dx=\int_0^{10}([x-10]+|[x-10]|)dx=0$

Now, $\int_0^{10}f^2(x)dx=\int_0^{10}([x]-10)^2dx$

$=\int_0^1 10^2dx+\int_1^2 9^2dx+\int_2^3 8^2dx+\cdots+\int_9^{10}1dx$

$=\left(1^2+2^2+3^2+\cdots+10^2\right)=\frac{10\times11\times21}{6}=385$

Q.41. The value of $\int_0^2\left(|2x^3-3x|+\left[x-\frac{1}{2}\right]\right)dx$, where [.] is greatest integer function is:

A) $\frac{7}{6}$

B) $\frac{19}{12}$

C) $\frac{17}{4}$

D) $\frac{3}{2}$

Answer: $\frac{17}{4}$

Solution: $\int_0^2|2x^3-3x|dx+\int_0^2\left[x-\frac{1}{2}\right]dx$

$=\int_0^{\sqrt{\frac{3}{2}}}\left(3x-2x^3\right)dx+\int_{\sqrt{\frac{3}{2}}}^{2}\left(2x^3-3x\right)dx+\int_0^{\frac{1}{2}}\left[x-\frac{1}{2}\right]dx+\int_{\frac{1}{2}}^{\frac{3}{2}}\left[x-\frac{1}{2}\right]dx+\int_{\frac{3}{2}}^{2}\left[x-\frac{1}{2}\right]dx$

$=\left[\frac{3x^2}{2}-\frac{2x^4}{4}\right]_0^{\sqrt{\frac{3}{2}}}+\left[\frac{2x^4}{4}-\frac{3x^2}{2}\right]_{\sqrt{\frac{3}{2}}}^{2}+\left(-\frac{1}{2}\right)+0+\left(\frac{1}{2}\right)$

$=\frac{9}{4}-\frac{9}{8}+8-6-\frac{9}{8}+\frac{9}{4}=2+\frac{9}{4}=\frac{17}{4}$

Q.42. The domain of $f(x)=\sin^{-1}\left[2x^2-3\right]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$ is

A) $\left(1, \frac{5-\sqrt{5}}{2}\right)$

B) $\left(1, \sqrt{\frac{5}{2}}\right)$

C) $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$

D) $(1, 4)$

Answer: $\left(1, \frac{5-\sqrt{5}}{2}\right)$

Solution: Given, $f(x) = \sin^{-1}\left[2x^2 - 3\right] + \log_2\left(\log_{\frac{1}{2}}\left(x^2 - 5x + 5\right)\right)$

Now defining domain of $\sin^{-1}$ we get,

$\because \left[2x^2 - 3\right] = -1, 0$ or 1

$\Rightarrow 2x^2 - 3 \in [-1, 2) \Rightarrow 2x^2 \in [2, 5) \Rightarrow x^2 \in \left[1, \frac{5}{2}\right)$

$\Rightarrow x \in \left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right]$

Now defining domain of $\log$ function we get, $\log_{\frac{1}{2}}\left(x^2 - 5x + 5\right) > 0$

$\Rightarrow 0 < x^2 - 5x + 5 < 1$

$\Rightarrow x \in \left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right]$

$\Rightarrow x^2 - 5x + 5 > 0 \Rightarrow x \in \left(-\infty, \frac{5-\sqrt{5}}{2}\right) \cup \left(\frac{5+\sqrt{5}}{2}, \infty\right)$

Also

$x^2 - 5x + 5 < 1 \Rightarrow x^2 - 5x + 4 < 0 \Rightarrow x \in (1, 4)$

$\Rightarrow x \in \left(1, \frac{5-\sqrt{5}}{2}\right) \cup \left(\frac{5+\sqrt{5}}{2}, 4\right)$

$\therefore \ x \in \left(1, \frac{5-\sqrt{5}}{2}\right)$

Q.43. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$ makes and angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of line are:

A) $\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

B) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

C) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$

D) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$

Answer: $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

Solution: Given, line of intersection of plae $ax + by = 3$ and $ax + by + cz = 0$ makes 30° angle with plane $y - z + 2 = 0$

Now finding direction ratios of line of intersection $(b, -a, 0)$

As angle between this line and $y - z + 2 = 0$ is 30°

So, $\sin\theta = \left|\frac{a}{\sqrt{a^2+b^2}.\sqrt{2}}\right| = \frac{1}{2}$

$\Rightarrow \ a^2 = b^2$

$\therefore$ Possible combination is $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

Q.44. Let n^{th} term of any sequence is given by $T_n = \frac{-1^3+2^3-3^3+4^3+\cdots+(2n)^3}{n(4n+3)}$, then $\sum_{n=1}^{15} T_n$ is equal to

A) 120

B) 200

C) 156

D) 121

Answer: 120

Solution: Given, $T_n = \frac{-1^3+2^3-3^3+4^3+\cdots+(2n)^3}{n(4n+3)}$

Now on rearranging we get,

$$T_n = \frac{2\left[2^3+4^3+\cdots+\left(2n^3\right)\right]-\left[1^3+2^3+3^3+\cdots+(2n)^3\right]}{n(4n+3)}$$

$$T_n = \frac{16\left(\frac{n(n+1)}{2}\right)^2-\left(\frac{2n(2n+1)}{2}\right)^2}{n(4n+3)}$$

$$= \frac{n^2(4n+3)}{n(4n+3)} = n$$

$$\therefore \sum_{n=1}^{15} T_n = \frac{15\times16}{2} = 120$$

Q.45. Let $\frac{1+i\sin\alpha}{1-2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real, where $\alpha,\beta\in[\pi,2\pi]$ and $z=\sin 2\alpha+i\cos 2\beta$, then $\sum\left(iz+\frac{1}{i\bar{z}}\right)$ is equal to

A) 1

B) 3

C) 2

D) 0

Answer: 1

Solution: Given, $\frac{1+i\sin\alpha}{1-2i\sin\alpha}$ is purely imaginary, so

$1-2\sin^2\alpha=0 \Rightarrow \sin^2\alpha=\frac{1}{2} \Rightarrow \alpha=\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

Similarly, $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real, so

$\Rightarrow \cos\beta=0 \Rightarrow \beta=\frac{3\pi}{2}$

$\Rightarrow \alpha=\frac{5\pi}{4}$ and $\frac{7\pi}{4} \Rightarrow \beta=\frac{3\pi}{2}$

Now, $z=\sin 2\alpha+i\cos 2\beta \Rightarrow z=1-i$ or $-1-i$

$\because \frac{1}{iz}=-\frac{iz}{|z|^2}=-\frac{iz}{2}$

So, $\sum\left(iz+\frac{1}{iz}\right)=\sum\frac{iz}{2}=\frac{i}{2}\sum z=\frac{i}{2}(-2i)=1$

Q.46. Let PQ be a building of height $10\ \mathrm{m}$ and angle of elevation of point P from a point A on the ground is 45°. Let B be another point from where foot of perpendicular on ground is R and angle of elevation of B from A is 30°. If angle of elevation of P from B is 60°. Then area of trapezium $PQRB$ and length AB is

A) $25, \frac{15\left(\sqrt{3}-1\right)}{2}$

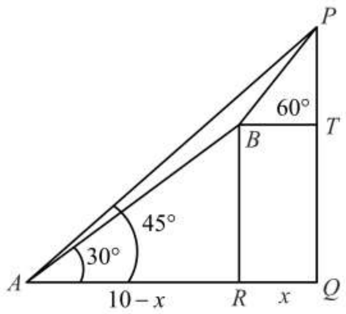
B) $15\left(\sqrt{3}+1\right), \frac{5\left(\sqrt{3}+1\right)}{2}$

C) $25, \frac{5\left(\sqrt{3}+1\right)}{2}$

D) $15\left(\sqrt{3}+1\right), \frac{15\left(\sqrt{3}-1\right)}{2}$

Answer: $25, \frac{15\left(\sqrt{3}-1\right)}{2}$

Solution: The figure as per the given data will be



In triangle APQ,

$AQ = PQ \cot 45^\circ = 10$

Let $QR = x$ and

$AR = 10 - x$

In triangle ABR,

$BR = AR \tan 30^\circ = \frac{10-x}{\sqrt{3}}$

i.e. $QT = \frac{10-x}{\sqrt{3}}$

Also in triangle PBT,

$PT = x \tan 60^\circ = \sqrt{3}x$

$\because QT + PT = 10$

So, $\frac{10-x}{\sqrt{3}} + \sqrt{3}x = 10 \Rightarrow 10 + 2x = 10\sqrt{3}$

$\Rightarrow x = 5\left(\sqrt{3} - 1\right)$

i.e. $BR = \frac{10-5\left(\sqrt{3}-1\right)}{\sqrt{3}} = 5\left(\sqrt{3} - 1\right)$

Hence, area of trapezium $PQRB$

$= \frac{1}{2}\left(5\left(\sqrt{3} - 1\right)\right)\left(10 + 5\left(\sqrt{3} - 1\right)\right)$

$= \frac{25}{2}\left(\left(\sqrt{3} - 1\right)\right)\left(1 + \sqrt{3}\right)$

$= 25$

Now, $AB = AR \cos 30^\circ = \frac{\sqrt{3}\left(10-5\left(\sqrt{3}-1\right)\right)}{2} = \frac{15\sqrt{3}-15}{2}$

Q.47. Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors such that $\vec{a} \times \vec{b} = 4\vec{c}, \vec{b} \times \vec{c} = 9\vec{a}, \vec{c} \times \vec{a} = \alpha\vec{b}$ and $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = 36$, then α is equal to

A) 36

B) 30

C) 26

D) 18

Answer: 36

Solution: Let a, b, c be the modulus of $\vec{a}, \vec{b}$ & $\vec{c}$ respectively.

Given, $\vec{a} \times \vec{b} = 4\vec{c}, \vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$

$\Rightarrow \vec{a}, \vec{b}$ & $\vec{c}$ are mutually perpendicular vectors.

$\because \left|\vec{a} \times \vec{b}\right| = 4\left|\vec{c}\right| \Rightarrow ab = 4c$

Similarly, $bc = 9a \,..(1)$ and $ca = \alpha b \,.....(2)$

Now multiplying equation (1) & (2) we get,

$abc^2 = 9a\alpha b \Rightarrow abc = 36\alpha$

Then, $c = 3\sqrt{\alpha}, a = 2\sqrt{\alpha}$ and $b = 6$

Now, $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = 36$

$\Rightarrow a + b + c = 36 \Rightarrow 5\sqrt{\alpha} + 6 = 36$

$\Rightarrow \quad \alpha = 36$